

# The domain wall fermion chiral condensate in quenched QCD

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We examine the chiral limit of domain wall fermions in quenched QCD. One expects that in a quenched simulation, exact fermion zero modes will give a divergent,  $1/m$  behavior in the chiral condensate for sufficiently small valence quark masses. Unlike other fermion formulations, domain wall fermions clearly demonstrate this behavior.

## 1. INTRODUCTION

It is a common assertion that quenched lattice QCD qualitatively produces many of the features found in continuum full QCD. However, a quenched simulation is expected to include topological gauge configurations which produce exact fermion zero modes. Not suppressed by the missing fermion determinant, such modes will cause divergent behavior in the small mass limit of the chiral condensate.

Surprisingly, the standard lattice fermion formulations do not demonstrate divergent behavior for the quenched chiral condensate at current lattice spacings and quark masses. One may imagine that the small eigenvalue spectrum of those lattice Dirac operators bears little resemblance to the continuum Dirac spectrum until the lattice spacing is substantially smaller. See the introduction of [1] for further discussion and references.

Starting from Kaplan's initial ideas for domain wall fermions (DWF) [2], several fermion formulations have been developed which use an infinite number of massive fermion flavors to produce an effective chiral-invariant action for a single massless fermion flavor. Using the overlap formalism [4], which describes the infinite flavor case, Narayanan and Neuberger demonstrated that these theories have a unique, integer valued index for the associated Dirac operator. Stud-

ies have shown that nearly exact zero modes persist in truncated theories, provided the number of heavy flavors is large enough. One should then expect to see these zero mode effects in quenched simulations.

## 2. THE CONTINUUM PICTURE

Using a spectral decomposition of the continuum Dirac operator, we can express the quark chiral condensate as a function of the quark mass for any ensemble of gauge fields

$$\begin{aligned}\langle\bar{\psi}\psi\rangle &= \frac{1}{V} \int d^4x \langle\bar{\psi}(x)\psi(x)\rangle \\ &= \frac{N_{\text{zm}}}{m} + 2m \int_0^\infty d\lambda \frac{\rho(\lambda)}{\lambda^2 + m^2},\end{aligned}\quad (1)$$

where we have used the  $\gamma_5$  symmetry of the Dirac spectrum and  $N_{\text{zm}}$  are the number of zero eigenvalues in the spectrum. We can similarly express the integrated pion correlator

$$\frac{1}{V} \int d^4x \langle\pi(x)\pi(0)\rangle = \frac{N_{\text{zm}}}{m^2} + 2 \int_0^\infty d\lambda \frac{\rho(\lambda)}{\lambda^2 + m^2}. \quad (2)$$

Spontaneous chiral symmetry breaking is related to a non-zero eigenvalue density at  $\lambda = 0$  by the Banks-Casher relation which, for the case of non-zero  $N_{\text{zm}}$ , becomes

$$\langle\bar{\psi}\psi\rangle \sim \frac{N_{\text{zm}}}{m} + \pi\rho(0) + \mathcal{O}(m), \quad m \rightarrow 0. \quad (3)$$

Finally, we note that the identity

$$m \int d^4x \langle\pi(x)\pi(0)\rangle = \langle\bar{\psi}\psi\rangle \quad (4)$$

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follows from the continuum chiral symmetry as represented by equations (1) and (2).

### 3. DOMAIN WALL FERMIONS

Our numerical simulations were performed using the boundary fermion variant of domain wall fermions [5]. Here one adds a fifth dimension to the normal 4-D space time of extent  $L_s$ . The gauge fields are not changed by this extension and do not depend on  $s$ . There are also two mass parameters for the fermions:  $m_f$ , the explicit four-dimensional bare quark mass and  $m_0$ , the five-dimensional bare quark mass. Except for different symbols for the three parameters listed above, our conventions follow [5].

In the free field case, the effective mass of the low energy state is [6]:

$$m_{\text{eff}}^{(\text{free})} = m_0(2 - m_0) [m_f + (1 - m_0)^{L_s}]. \quad (5)$$

where, for one flavor physics,  $m_0$  should be in the range (0, 2). This simple equation reveals two important features of domain wall fermions. First, as  $L_s \rightarrow \infty$ ,  $m_{\text{eff}} \propto m_f$  and the proportionality constant depends on the five-dimensional mass  $m_0$ . Second, for finite  $L_s$ , the Wilson-like additive correction to  $m_{\text{eff}}$  is exponentially suppressed in  $L_s$  and the rate of decay is determined again by  $m_0$ . In the interacting case, we expect that the effective mass will behave in a similar fashion where the residual mass term  $m_{\text{res}}$  should vanish exponentially in  $L_s$  at a rate governed by  $m_0$  and the coupling constant. Indeed, numerical studies [6] indicate that the decay remains exponential with a decay rate that becomes faster as the continuum limit is approached.

### 4. QUENCHED DWF AT ZERO TEMPERATURE

We begin by examining the extent to which domain wall fermions reproduce the effects of chiral symmetry for finite  $L_s$  in a quenched simulation by computing the ratio of the two sides of (4) on  $8^3 \times 32$  lattices at  $\beta = 5.7$  and 5.85. We choose to work at a domain wall height,  $m_0 = 1.65$ . This choice is supported by numerical studies of the chiral condensate for several values of  $m_0$ ,  $L_s$  and  $m_f$  at these values of  $\beta$ .

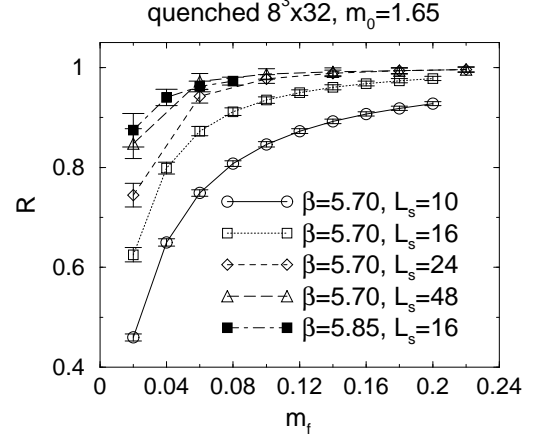


Figure 1. Ratio  $R$  of  $m \int d^4x \langle \pi(x) \pi(0) \rangle$  to  $\langle \bar{\psi} \psi \rangle$

In Figure 1 we show the ratio of quantities proportional to the left and right hand sides of (4). The constant,  $m_f$ -independent, behavior of this ratio, required by chiral symmetry, is increasingly visible in Figure 1, for the curves with larger  $L_s$ . Particularly striking is the comparison of this ratio between  $\beta = 5.7$  and 5.85. The weaker coupling with  $L_s = 16$  shows a degree of chiral symmetry present at the stronger coupling only for a three times larger  $L_s$ . This is consistent with the picture that stronger coupling requires larger  $L_s$  [6].

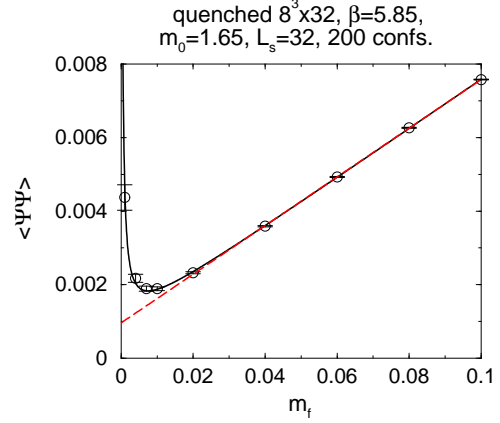


Figure 2.  $1/m_f$  divergence in  $\langle \bar{\psi} \psi \rangle$

Next we study the small mass limit of  $\langle \bar{\psi} \psi \rangle$  computed on 200 quenched configurations at  $\beta = 5.85$  and  $L_s = 32$ . The results, shown in Figure 2, show a clear signal for a  $1/m_f$  divergence for  $m_f < 0.01$ . The solid line is a fit for  $0.001 \leq m_f \leq 0.1$  which includes a  $1/m_f$  term and the dashed line is a linear fit for  $0.01 \leq m_f \leq 0.1$ .

The constant and linear terms are consistent between these two fits within errors and the  $\chi^2/\text{dof}$  is one for both fits. Table 1 summarizes the rest of our  $T = 0$  results. We have used  $m_0 = 1.65$  for all simulations.

We believe that this is the first time that the  $1/m_f$  behavior in  $\langle\bar{\psi}\psi\rangle$  expected from fermion zero modes present in quenched calculations has been seen. Notice the decrease in the strength of this term, shown in Table 1, when the volume is increased from  $8^3 \times 32$  to  $16^3 \times 32$ .

Table 1

Fit to  $\langle\bar{\psi}\psi\rangle = c_{-1}/m_f + c_0 + c_1 m_f$ 

	$8^3 \times 32$	$8^3 \times 32$	$16^3 \times 32$
$\beta = 5.70$	$\beta = 5.70$	$\beta = 5.85$	$\beta = 5.85$
$L_s = 48$	$L_s = 48$	$L_s = 32$	$L_s = 32$
40 configs	200 configs	90 configs	
$c_{-1}(\times 10^{-6})$	1.6(4)	3.8(3)	0.60(9)
$c_0(\times 10^{-3})$	2.15(3)	0.82(3)	0.89(1)
$c_1(\times 10^{-2})$	6.01(2)	6.73(3)	6.71(2)
$\chi^2/\text{dof}$	8.99	1.24	2.96

## 5. QUENCHED DWF AT FINITE TEMPERATURE

In Table 2 we summarize a quenched study of  $\langle\bar{\psi}\psi\rangle$  on a  $16^3 \times 4$  lattice evaluated for  $\beta$  slightly above  $\beta_c = 5.6925$ . Note, we continue to see a divergent behavior of  $\langle\bar{\psi}\psi\rangle$ . To our surprise, we also see a non-zero constant contribution to the chiral condensate well into the symmetric phase. Figure 3 shows the behavior of both  $c_{-1}$  and  $c_0$  as we increase  $\beta$  above  $\beta_c$ . Each shows the rapid decrease as the system heats up expected if their origin is topology-induced fermionic, zero modes. This never-before-seen, quenched small-mass behavior of  $\langle\bar{\psi}\psi\rangle$ , suggests that domain wall fermions offer new insights into the continuum behavior of quenched QCD.

## REFERENCES

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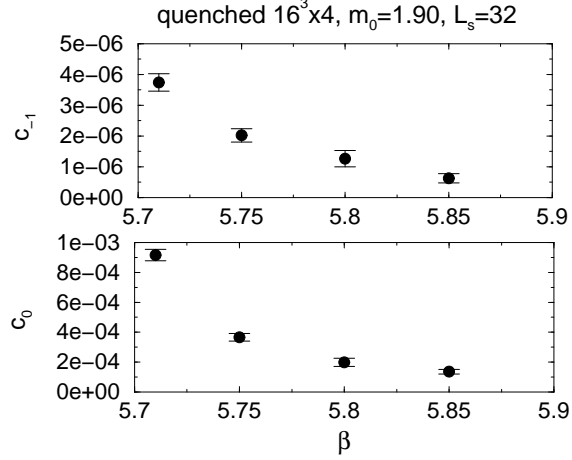
Figure 3. Fit coefficients for  $T > T_c$ 

Table 2

 $16^3 \times 4, m_0 = 1.90, L_s = 32$ 

$\beta, \text{conf}$	$c_{-1}$ ( $\times 10^{-6}$ )	$c_0$ ( $\times 10^{-3}$ )	$c_1$ ( $\times 10^{-2}$ )	$\chi^2/\text{dof}$
5.71, 128	3.7(3)	0.92(4)	8.97(5)	5.34
5.75, 116	2.0(2)	0.37(3)	9.14(3)	1.44
5.80, 84	1.3(3)	0.20(3)	9.11(3)	0.30
5.85, 136	0.6(1)	0.14(2)	9.02(2)	0.81

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